
ECONOMIC DEVELOPMENT AND SUSTAINABILITY IN AN AGGREGATIVE MODEL INCORPORATING THE ENVIRONMENT

Binh Tran-Nam
Australian Taxation Studies Program (ATAX)
University of New South Wales

Abstract

We investigate an infinite-horizon aggregative closed economy in which production depends essentially on physical capital, natural capital and labour. The natural capital stock is modelled as a renewable resource. The change in the stock of natural capital depends on its autonomous evolution, production and consumption externalities, and environment maintenance programs. The economy is shown to be sustainable only if, for any marginal propensity to consume (MPC), the rate of taxation to maintain the environment exceeds a critical level, or equivalently, for any tax rate, the MPC is below a critical value. If human activities have a net beneficial effect on the environment, the economy will converge to a unique and stable steady state, which can be viewed as a generalized Solow-Swan balanced path. The condition for such a steady state to be sustainable is derived. In a sustainable steady state, the tax rate and MPC can be chosen to maximize per capita consumption.

Introduction

It is now generally acknowledged that the aggregate output of an economy and, hence, its economic development path, depend ultimately on how it make uses of its physical, human, social and environmental capital. The accumulation of physical capital as a determinant of economic growth features prominently in the Solow-Swan model (see Solow, 1956; Swan, 1956). This model provides simple testable propositions about how the exogenously given rates of saving, population growth and technological innovation influence the steady-state level of per capita income

More recently, Mankiw, Romer and Weil (1992) incorporated human capital into the textbook Solow-Swan model and demonstrated that the augmented model provides an excellent explanation of the international variation in the standard of living. In a separate development, economists have turned increasingly to endogenous-growth models which go beyond the Solow-Swan model to argue for externalities in the generation of both human capital and technological innovations, and for profit as the driving force behind innovation. Various strands of the endogenous growth literature, which dates back to the work of Uzawa (1965), are summarized in Romer (1994).

Social capital, alternatively called social capabilities, is a more elusive concept, which does not render itself easily to formal analysis. It includes such factors as openness and competitiveness of the economy, institutional arrangements, secure property rights, honesty, trust and interpersonal networks. In short, social capital represents a set of any intangible things that reduce transaction costs and, thus, help markets operate more smoothly. The role of social capital as an input in the production process has been considered mainly by development economists in connection with developing or transitional economies (see, for example, Hasson and Henrekson, 1994).

The natural capital stock refers to the totality of all ecosystems, which include, for example, water, soil, forest cover, the atmosphere, minerals, ores and fossil fuels. While a complete theory of the mining firm was first formulated by Hotelling (1931) more than sixty years ago, economists only began constructing general equilibrium models to accommodate resource exhaustibility in the early 1970s. This effort, triggered by the Report for the Club of Rome (see Meadows et al., 1972) and the oil shocks, has resulted in a substantial expansion of the literature on exhaustible resources (see, for example, Dasgupta and Heal, 1979; Kemp and Long, 1980). A large part of this vein of literature is concerned with the optimal depletion of nonrenewable resources.

More recently, research on the environment has focused on biodiversity and renewable natural resources (see, for example, Pearce and Turner, 1990; Perrings, 1994; Dasgupta, 1996). The major theoretical issues in this new wave of literature include intergenerational incidence of costs and benefits, causes of environmental degradation, valuation of environmental resources, property rights and economic instruments, and international governance and the global commons. Sustainable development at the aggregate level appears to involve four major issues:

absorption of the society's wastes, exhaustion of nonrenewable resources, preservation of ecosystems and reduction of environmental amenity. Of particular interest is the proposed 'green' GDP, which deducts, among other things, depreciation of environmental resources from GDP.

The present paper is motivated by the following observation. In recent years, there has been a powerful revival of the Solow-Swan model in the macroeconomic literature. In fact, popular intermediate macroeconomic textbooks almost uniformly start with some variants of the Solow-Swan model of long-run growth (see, for example, Hall and Taylor, 1997; Mankiw, 1997; Romer, 1996). As noticed by Dasgupta (1996), there is no mention of environmental resources. The implicit assumption is that natural resources are neither scarce now nor scarce in the future. This kind of supposition is undesirable.

This paper seeks to address this omission by treating natural capital as an essential factor of production. There is an illusory distinction between resource and environmental economists. Resource economists, who are interested in population ecology, characterize complex systems by the population sizes (or, alternatively tonnage) of different species. Environmental economists, who are interested in systems ecology, typically summarize complex systems in terms of indices of 'quality' of air, soil or water. This paper combines both approaches and treats the environmental capital as a stock measurable in some constant quality units. Since this paper focuses on economic theory, all practical problems associated with measuring natural capital are assumed away.

The stock of the environment changes in much the same way as a stock of manufactured capital. Unperturbed by human economic activities, the environmental stock grows or decays autonomously over time. Both production and consumption of the final output can degrade the environment (thus depleting its stock of quality units) while collective environmental programs, funded by income tax revenue, can repair, maintain or even improve the environment. In treating environmental damages as reversible, the paper identifies environmental resources with renewable natural resources. The possibility of investment in natural capital has been suggested by Pearce and Turner (1990), and John and Pecchenino (1994).

The remainder of this paper is organized as follows. Section 2 presents the formal model. In Section 3, the concept of sustainability is discussed and the set of sustainable MPCs (or tax rates) for any given tax

rate (or MPC) derived. Section 4 demonstrates that, under suitable conditions, the economy converges to a unique and stable balanced-path steady state, and obtains the condition for such a steady state to be sustainable. In Section 5, the optimal choice of the tax rate and the MPC is explored. Some concluding remarks, including some suggestions for possible extensions, are then given in the final section.

The model

Consider a continuous-time, infinite-horizon, closed economy, which produces a single final good with the aid of physical capital, environmental resources and labour. Let $Y(t)$, $K(t)$, $E(t)$ and $L(t)$ denote gross output, physical capital stock, natural capital stock and labour at time t , respectively. The aggregate production function is then written as

$$Y(t) = F[K(t), E(t), L(t)] \quad (1)$$

where all inputs are essential, F exhibits constant returns to scale, and all marginal products are positive but diminishing. It is further assumed that the marginal product of the i -th factor approaches infinity (zero) as the amount of that input approaches zero (infinity).

The output of the final good can be consumed, saved as capital or spent to maintain or improve the environment. The national accounting identity can be written as

$$Y(t) \equiv C(t) + S(t) + T(t) \quad (2)$$

where $C(t)$, $S(t)$ and $T(t)$ stand for aggregate consumption, saving and tax at time t , respectively. It is further assumed that tax revenue is a constant fraction of output and that consumption is a constant fraction of disposable income. Thus,

$$T(t) = \tau Y(t) \quad (3)$$

and

$$C(t) = a[Y(t) - T(t)] \quad (4)$$

where τ ($0 < \tau < 1$) is the tax rate and a ($0 < a < 1$) is the marginal propensity to consume (MPC).

The labour force is assumed to grow at the exogenously given, positive constant rate n (> 0) over time, i.e.

$$\dot{L}(t) = nL(t) \quad (5)$$

where $L_0 = L(0) > 0$. The stock of capital depreciates naturally over time at the exponential rate δ ($0 < \delta < 1$) and all saving is invested in capital formation. Thus, net investment in capital at time t can be described by

$$\dot{K}(t) = S(t) - \delta K(t) \quad (6)$$

where $K_0 = K(0) > 0$.

The instantaneous rate of change of the environmental stock (measured in some constant quality unit) is determined linearly by three forces. In the absence of human economic activity, the stock of the environment changes naturally over time at the exponential rate α . (The parameter α may be positive, zero or negative according as the environment grows, remains unchanged or decays autonomously over time.) The production of the final good causes external damages to the environment, thus depleting β units the environmental stock for every unit of the final good produced. Similarly, each unit of the final good consumed depletes γ units of the environmental stock. Finally, environmental programs, funded by the entire tax revenue, generate ϕ units of the environmental stock per unit of the tax spent. Assuming that the government runs a balanced budget at any instant of time, the evolution of the environment over time can thus be described by

$$\dot{E}(t) = \alpha E(t) + \phi T(t) - \beta Y(t) - \gamma C(t) \quad (7)$$

where $E_0 = E(0) > 0$ and α (> -1) is assumed to be smaller than n . The assumption $\alpha < n$ means that population grows at a faster rate than the natural environment. The assumption $\alpha > -1$ means that if the environment decays autonomously, its decay is sufficiently slow so that some positive stock of the environment always exists at any instant of time. Without loss of generality, taxation revenue is assumed to be costlessly collected and government failures to be non-existent so that the

entire tax revenue can be spent on the environment. It also seems reasonable to assume that $\phi > \beta$ because production externalities are typically unintentional whereas environmental actions are well planned and executed.

The first step is to reformulate the model in per worker terms. Recalling the linear homogeneity of F , gross output can be written in per worker terms as follows:

$$y(t) = f[k(t), e(t)] \quad (1')$$

where $y(t) (\equiv Y(t)/L(t))$, $k(t) (\equiv K(t)/L(t))$ and $e(t) (\equiv E(t)/L(t))$ are output, capital and environmental stock per worker at time t , respectively. From the assumed properties of F , the function f exhibits the following characteristics $f[0, e(t)] = f[k(t), 0] = 0$, $f_k > 0$, $f_e > 0$, $f_{kk} < 0$, $f_{ee} < 0$, $f_k \rightarrow \infty$ (0) as $k \rightarrow 0$ (∞) and $f_e \rightarrow \infty$ (0) as $e \rightarrow 0$ (∞).

The equations describing the dynamic evolution of the economy are (5), (6) and (7). Combining (2)-(5) and (1'), equation (6) can be rewritten in terms of the capital-labour ratio as

$$\dot{k}(t) = (1-a)(1-\tau)f[k(t), e(t)] - (\delta+n)k(t) \quad (6')$$

where $k_0 \equiv K_0/L_0$. Similarly, using (1)-(5) and (1'), equation (7) can be expressed in per worker terms as

$$\dot{e}(t) = [(\phi+\gamma a)\tau - (\beta+\gamma a)]f[k(t), e(t)] - (n-\alpha)e(t) \quad (7)$$

where $e_0 \equiv E_0/L_0$.

Long-run Sustainability Conditions

As natural capital is an essential input in the production process, the prosperity (and ultimately the survival) of the economy depends, among other things, on its ability to manage the environment. In particular, it is unwise for an economy to develop by running down the environment indefinitely. It is possible to distinguish between two concepts of sustainability: short-run sustainability (finite-time horizon) and long-run (infinite-time horizon) sustainability. This section is concerned mainly with conditions for long-run sustainability.

Suppose that the economy's planning time horizon is M . The economy is then said to be sustainable in the short run if, at any finite time $t \leq M$, per capita consumption $c(t) (\equiv C(t)/L(t))$ exceeds a given subsistence consumption level $\bar{c} (> 0)$. This condition can be written in terms of output per worker as follows:

$$f[k(t), c(t)] \geq d[a(1-\tau)] \quad t \leq M \quad (8)$$

In this case, short-run sustainability simply requires that output per worker exceeds a positive constant any instant of time. This in turn requires that $k(t)$ and $c(t)$ be both at least positive.

The survival of the economy depends not only on consumption but also on the environment. Life can only be sustained if human beings enjoy a sufficient amount of environment (in some constant quality units). If one is willing to think of the environment as a private good (no joint consumption) then short-run sustainability also requires that:

$$e(t) \geq \bar{e} \quad t \leq M \quad (9)$$

where $\bar{e} (> 0)$ stands for the subsistence per capita environmental level as dictated by human physiology. However the condition (9) will not be insisted upon as it does not seem appropriate to treat the environment as a pure private good. Unlike consumption, which is a private good, the environment exhibits some public-good properties.

The condition (8) is short sighted for an infinitely-lived economy. An economy is said to be sustainable in the long run if

$$f[k(t), c(t)] \geq d[a(1-\tau)] \quad t \geq 0 \quad (8')$$

The condition (8') implies that, if a steady-state per capita consumption exists, then it must be equal to or greater than $\bar{c}$. There is no guarantee that the environment stock will remain positive as time grows indefinitely large. This depends crucially on the sign of $(\phi+\gamma\alpha)\tau-(\beta+\gamma\alpha)$. Let consider the various cases separately:

Case 1: $(\phi+\gamma\alpha)\tau-(\beta+\gamma\alpha) = 0$

Let us consider the long-run evolution of an economy in which $(\phi+\gamma\alpha)\tau-(\beta+\gamma\alpha) = 0$, i.e. human activities have zero net effect on the environment in every time period. In this case, equation (7') gives rise to

$$e(t) = e_0 \exp[-(n-\alpha)t] \quad (10)$$

Since $(n-\alpha) > 0$ by assumption, $e(t)$ will steadily approach zero from above as time tends to infinity. As $e(t)$ becomes smaller and smaller exponentially, the curve $(1-a)(1-\tau)f[k(t), e(t)]$ will eventually lie below $(\delta+n)k(t)$ so that $\dot{k}(t)$ will also become negative, as dictated by equation (6'). This in turn means that $k(t) \rightarrow 0$ as $t \rightarrow \infty$. The evolution of the economy in this case is graphically illustrated in the following graph.

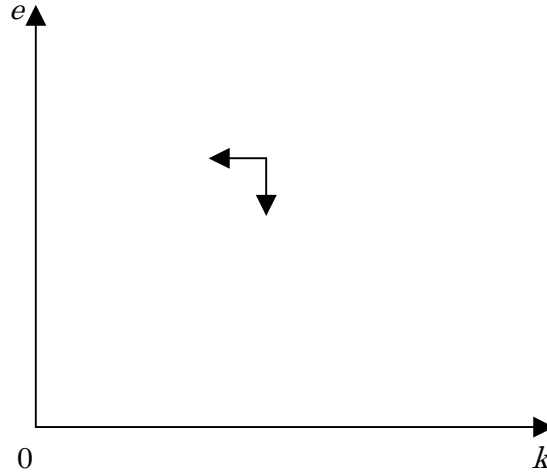


Figure 1. An unsustainable long run ($\tau = (\beta+\gamma\alpha)/(\phi+\gamma\alpha)$)

Since $e(t)$ and $k(t)$, and thus $f(t)$ and $\alpha(t)$, all approach zero as t becomes indefinitely large, that the economy is unsustainable in the long run.

Case 2: $(\phi+\gamma\alpha)\tau-(\beta+\gamma\alpha) < 0$

From equation (7'), if $(\phi+\gamma\alpha)\tau-(\beta+\gamma\alpha) < 0$, i.e. human activities have a net negative impact on the environment in every time period, then $\dot{e}(t) < 0$ for all $t \geq 0$. It can then be shown that $e(t) < e_0 \exp[-(n-\alpha)t]$ for all $t \geq 0$. The above argument again applies.

We can now state the following proposition.

Proposition 1

If human activities have a net zero or negative effect on the environment, the economy is unsustainable in the long run in the sense that physical and natural capital per worker (and thus per capita output and consumption) will tend to zero as time grows indefinitely large.

Note that in the case $(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha) < 0$ it is conceivable that the finite-time condition (8) may not be satisfied, i.e. the economy may not even be sustainable in the short run.

Proposition 1 implies that a necessary condition for the economy to be sustainable in the long run is that $(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha) > 0$. Treating the tax rate as given, the condition $(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha) > 0$ is equivalent to $a < (\phi\tau - \beta)/[\gamma(1 - \tau)]$. Bearing in mind that both tax rate and MPC must lie within the open interval $(0, 1)$, it can be seen that if the tax rate is too small (i.e. $\tau \leq \beta/\phi$) then no positive MPC is sustainable, and if the tax rate is sufficiently large (i.e. $\tau \geq (\beta + \gamma)/(\phi + \gamma)$) then a sustainable MPC can take any value in the interval $(0, 1)$. Alternatively, treating the consumption rate as given, the condition $(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha) > 0$ is equivalent to $\tau > (\beta + \gamma\alpha)/(\phi + \gamma\alpha)$.

The above results can be summarized as follows:

Proposition 2

- For any given tax rate,
 - (i) if $\tau \leq \beta/\phi$ then no positive MPC is sustainable
 - (ii) if $\beta/\phi < \tau < (\beta + \gamma)/(\phi + \gamma)$ then the set of sustainable MPCs is $(0, (\phi - \beta)/[\gamma(1 - \tau)]$;
 - (iii) $\tau \geq (\beta + \gamma)/(\phi + \gamma)$ then the set of sustainable MPCs is $(0, 1)$.
- For any given MPC, the set of sustainable tax rates is $[(\beta + \gamma\alpha)/(\phi + \gamma\alpha), 1)$.

It can be inferred from Proposition 2 that

- an increase (a decrease) in the tax rate in the relevant range widens (narrows) the choice of sustainable MPCs; and
- an increase (a decrease) in the MPC narrows (widens) the choice of sustainable tax rates.

These results are intuitively clear. If more (less) resources are spent to repair the environment, a larger (smaller) fraction of the remaining output is now available for consumption while keeping the economy sustainable. Alternatively, if a larger (smaller) fraction of output is consumed, then the range sustainable tax rates will become narrower (wider).

A Sustainable Steady State

Suppose now that $(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha) > 0$, i.e. human activities produce a net beneficial effect on the environment for every time period. The relevant question now is whether under that condition the per capita consumption will converge to a sustainable steady state? Let us start with a formal definition of a steady state.

Definition of a steady state

A steady state of a sustainable economy is a balanced path equilibrium in which the capital stock and the environmental stock both grow at the same rate as the labour force.

It is clear that this definition is a straightforward generalization of the balanced-path steady state in the neoclassical Solow-Swan model of economic growth. It follows that in a steady state, output per worker and per capita consumption are constant.

Existence and uniqueness of a steady state

A steady state of the economy, if it exists, is a pair (k^*, e^*) satisfying a system of two non-linear equations

$$(1-a)(1-\tau)f(k^*, e^*) - (\delta+n)k^* = 0 \quad (11)$$

$$[(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha)]f(k^*, e^*) - (n-\alpha)e^* = 0 \quad (12)$$

To solve for k^* , it is possible to eliminate e^* by making use of the fact that equations (11) and (12) together imply

$$e^* = Ak^* \quad (13)$$

where $A \equiv (\delta+n)[(\phi + \gamma\alpha)\tau - (\beta + \gamma\alpha)]/[(n-\alpha)(1-a)(1-\tau)] > 0$. This proportionality is intuitively clear. Since physical capital (K^*) and natural capital (E^*)

grow at the same rate in a balanced-path steady state, the ratio of k^* over e^* (ie, K^* over E^*) must necessarily be a constant.

Substituting (13) into (11) yields

$$(1-a)(1-\tau)f(k^*, Ak^*) = (\delta+n)k^* \quad (14)$$

Let $g(k) \equiv f(k, Ak)$. Since $f(k, e)$ is increasing and concave in (k, e) , g is also increasing and concave in k . Apart from the trivial solution $k = 0$, equation (14) has a positive solution because $g(0) = 0$, $g' > 0$, $g' \rightarrow \infty$ as $k \rightarrow 0$ and $g' \rightarrow 0$ as $k \rightarrow \infty$. Further, this positive solution k^* is also unique because g is concave.

Stability of the unique steady state

From equation (11), it is clear that for any $e(t)$, if $k(t) < (>) k^*$ then $\dot{k}(t) > (<) 0$. Similarly, equation (12) implies that for any $k(t)$, if $e(t) < (>) e^*$ then $\dot{e}(t) > (<) 0$. This means that the steady state is globally asymptotically stable in the sense that the economy will always converge to (k^*, e^*) from any initial state (k_0, e_0) as time grows indefinitely large. This is graphically illustrated in Figure 2. It is not difficult to see that the trivial solution $(0, 0)$ to the system of equations (11) and (12) is unstable.

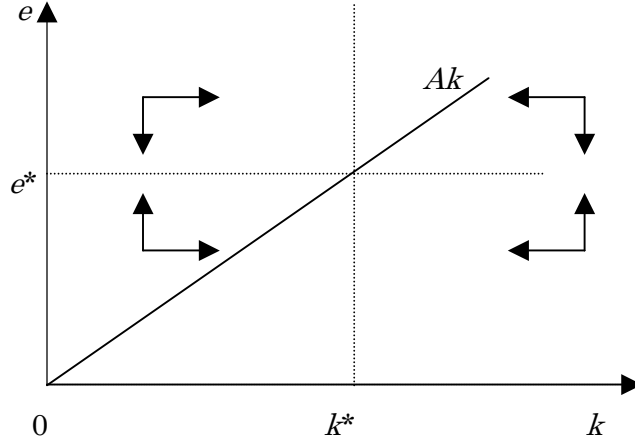


Figure 2. Stability of the steady state when $\tau > (\beta + \gamma a) / (\phi + \gamma a)$

To summarize, we may now state the following proposition.

Proposition 3

If human activities are overall beneficial to the environment, the economy converges from any initial conditions to a unique steady state in which manufactured capital, natural capital and labour all grow at the same rate.

Sustainable steady state

There is no guarantee that the steady state derived above is sustainable. The economy is sustainable in the long run only if k^* is sufficient large to produce the subsistence consumption level. In the steady state $(1-a)(1-\tau)f(k^*, e^*) = (\delta+n)k^*$. Combining this result and (8'), we can now establish the following proposition.

Proposition 4

The economy's steady state is sustainable if

$$k^* \geq (1-a)(\delta+n)\epsilon a \quad (15)$$

The above condition is likely to be satisfied since ϵ typically lies very close to zero. (In fact, in most studies, ϵ is conventionally assumed to be equal

to zero.) The inequality (15) will be assumed to hold true in the next section.

Optimal Sustainable Growth: Golden-Rule Steady State

The tax rate and MPC have so far been treated as exogenous to the model. We can now talk about optimal growth by viewing these rates as choice variables. Optimal growth can be introduced to a sustainable economy in two different ways. The first, and more formal, way is to assume that there is a long-lived government which chooses the tax rate and the consumption rate to maximize a social target function, defined as an indefinite integral of a time discounted instantaneous utility on per capita consumption. The existence of a far-sighted government is completely consistent with the assumption of homogeneous economic agents assumed in the model. The second, and simpler, way is to find the tax rate and consumption rate to maximize the per capita steady-state consumption. This paper follows the second approach and seeks to characterize the golden rule sustainable steady state in which per capita consumption is maximized.

The golden rule k^* , denoted by k^*_{gol} , and optimal pair of tax rate and consumption rate, denoted by τ^*_{gol} and a_{gol} respectively, can be derived by the following two-stage maximization procedure.

Stage one: For any sustainable tax rate τ^* , choose $k^*_{\text{gol}}(\tau)$ and $a_{\text{gol}}(\tau)$ to maximize c^* .

Keeping in mind that consumption is equal to disposable income minus gross investment in physical capital where gross investment matches depreciation exactly in a steady state, per capita consumption can be expressed as

$$c^* = (1-\tau)f(k^*, Ak^*) - (\delta+n)k^* = (1-\tau)g(k^*) - (\delta+n)k^* \quad (16)$$

where A , as defined in equation (13), depends on a and τ . Let $k^*_{\text{gol}}(\tau)$ be the value of k^* that maximizes c^* for any given sustainable τ . This value is determined by the first-order necessary condition

$$\partial c^* / \partial k^* = (1-\tau)(f_k + Af_e) - (\delta+n) = 0 \quad (17)$$

or, equivalently,

$$(1-\tau)g'(k^* | A) = \delta+n \quad (17')$$

where A now depends on a only. Equation (17) can be interpreted as requiring that the after-tax weighted sum of the marginal products of physical and natural capital be equal to the sum of the depreciation and population growth rates. Note also that $\partial^2 c^* / \partial k^{*2} < 0$ (since g is concave) so that the second-order condition for a maximum is also satisfied.

Since g' is strictly decreasing from ∞ to 0 as k increases from 0 to ∞ , equation (17') has a unique, positive solution given by

$$k^*_{\text{gol}}(\tau) = h[(\delta+n)/(1-\tau) | A] \quad (18)$$

where h is the inverse function of g' . It is now necessary to show that for any sustainable τ , there exists a unique consumption rate $a_{\text{gol}}(\tau)$ that gives rise to $k^*_{\text{gol}}(\tau)$. This can be done by recalling from equation (11) that $(1-a)(1-\tau)f(k^*, Ak^*) - (\delta+n)k^* = 0$ in a steady state. For any given τ , k^* decreases monotonically as a increases in the open interval $(0, 1)$. Further, as a approaches 0, k^* approach $\tilde{k}(\tau)$ where $\tilde{k}(\tau)$ is given by $(1-\tau)f(\tilde{k}, \tilde{A}\tilde{k}) - (\delta+n)\tilde{A} = 0$ and $\tilde{A} \equiv (\delta+n)(\phi\tau-\beta)/[(n-\alpha)(1-\tau)]$. Now, since $c^*_{\text{gol}}(\tau) = (1-\tau)f[k^*_{\text{gol}}(\tau), A_{\text{gol}}k^*_{\text{gol}}(\tau)] - (\delta+n)k^*_{\text{gol}}(\tau) > 0$, it is apparent that $k^*_{\text{gol}}(\tau) < \tilde{k}(\tau)$. Thus, there exists a unique $a_{\text{gol}}(\tau) > 0$ that satisfies

$$(1-a_{\text{gol}})(1-\tau)f(k^*_{\text{gol}}, A_{\text{gol}}k^*_{\text{gol}}) = (\delta+n) \quad (19)$$

where $A_{\text{gol}} \equiv (\delta+n)[(\phi+\gamma a_{\text{gol}})\tau - (\beta+\gamma a_{\text{gol}})]/[(n-\alpha)(1-a_{\text{gol}})(1-\tau)]$ and k^*_{gol} as specified in (18).

Stage two:

The calculations described above yield $c^*_{\text{gol}}(\tau) = c^*[\tau, k^*_{\text{gol}}(\tau), a_{\text{gol}}(\tau)]$ as a function of τ only. In principle, we can choose τ to maximize $c^*_{\text{gol}}(\tau)$. The solution to this maximizing problem exists since c is concave in τ . This procedure yields τ_{gol} and, thus, a_{gol} , k^*_{gol} and c^*_{gol} . The set of values $\{\tau_{\text{gol}}$,

$a_{\text{gol}}, k^*_{\text{gol}}, c^*_{\text{gol}}$ then summarizes the golden-rule steady state of the economy.

Conclusion

The paper presents a preliminary effort to incorporate natural capital into the neoclassical Solow-Swan model of long-run economic growth with no technological innovation. Environmental resources, measured in constant quality units, collectively represent an essential input in the production process. Natural capital is treated as renewable in the sense that damages done to the environment by production and consumption externalities are reversible and can be corrected by collective maintenance action, which is financed by income taxation. The evolution of the economy over time is then described by the continuous changes in per-worker physical capital and natural capital.

The economy is shown to be sustainable in the long run only if human activities have a nonnegative effect on the environment. For any given tax rate (or MPC), the set of sustainable MPCs (or tax rates) is derived. If human activities have a net zero effect on the environment, the economy is sustainable in the short run but not in the long run. If human activities have a net negative effect on the environment, the economy is unsustainable in an infinite time horizon and may even be unsustainable in a finite time horizon. If human activities have an overall beneficial effect on the environment then the economy will converge to a unique and stable steady state in which both physical and natural capital will grow at the same rate as labour.

The steady state of the economy has been analyzed by treating the tax rate and MPC as exogenous to the economy. In the presence of a far-sighted long-lived government, these rates can be chosen to maximize welfare. The existence of such a government is consistent with the implicit assumption of the representative agent in the model. In any sustainable steady state, it is shown that there exists a unique pair of tax rate and MPC that maximizes per capital consumption.

This preliminary research can be extended in several different directions. The first, and most obvious, extension is to consider the role of exogenous technological innovation or the accumulation of other kinds of capital, namely human and social capital, in the process of economic growth. Adding exogenous technical progress or endogenous human

capital to the model can be done in a conventional fashion, but modelling social capital would be far more problematic. The second and more substantial extension is to recognize that economic agents do not have the same time horizon as the economy. Since economic agents are short-lived, this calls for a time-discrete model incorporating overlapping generations with an explicit optimizing framework (see Tran-Nam and Truong, 2001). The third possible extension is to recognize that natural capital is not only an input to the production process, but also a direct source of utility in its own right (see Tran-Nam and Truong, 2001). As a source of pleasure, natural capital can be thought of as a public good. Finally, it may be worthwhile to endogenize the environmental-damaging parameters β and γ . This means to treat β (γ) as being dependent on the environment stock and output (consumption) level.

References

- Dasgupta, Partha. 1996. "The Economics of the Environment." *Environment and Development Economics*. 1(4): pp. 387-428.
- Dasgupta, Partha and Geoffrey M. Heal. 1979. *Economic Theory and Exhaustible Resources*. Cambridge: Cambridge University Press.
- Hall, Robert E. and John B. Taylor. 1997. *Macroeconomics*. New York: Norton.
- Hasson, Par and Magnus Henrekson. 1994. "What Makes A Country Socially Capable of Catching Up?" *Weltwirtschaftliches Archiv*. 130(4): pp. 760-83.
- Hotelling, Harold. 1931. "The Economics and Exhaustible Resources." *Journal of Political Economy*. 39(2): pp. 137-175.
- John, A. Andrew and Rowena A. Pecchenino. 1994. "An Overlapping Generations Model of Growth and the Environment." *Economic Journal*. 104(4): pp 1393-1410.
- Kemp, Murray C. and Ngo V. Long. 1980. *Exhaustible Resources, Optimality and Trade*. Amsterdam: North Holland.
- Mankiw, N. Gregory. 1997. *Macroeconomics*. New York: Worth.

Mankiw, N. Gregory, David Romer and David N. Weil. 1992. "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics*. 107(2): pp 407-437.

Meadows, Donella H., Dennis L. Meadows, Jørgen Randers and W.W. Behrens. 1972. *The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind*, London: Potomac Associates.

Pearce, David A. and R. Kerry Turner. 1990. *Economics of Natural Resources and the Environment*. Baltimore : Johns Hopkins University Press.

Perrings, Charles. 1994. *Biodiversity Loss: Ecological and Economic Perspectives*. Cambridge: Cambridge University Press.

Romer, David. 1996. *Advanced Macroeconomics*. New York: McGraw Hill.

Romer, Paul M. 1994. "The Origins of Endogenous Growth." *Journal of Economic Perspectives*. 8(1): pp 3-22.

Solow, Robert M. 1956. "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics*. 70(1): pp. 65-94.

Swan, Trevor W. 1956. "Economic Growth and Capital Accumulation." *Economic Record*. 32(63): pp. 327-368.

Tran-Nam, Binh and Cong Nghe Truong. 2001. "The Environment, Externalities and Sustainable Growth in An Overlapping Generations Model." In *Economic Theory and International Trade*. Eds. Alan Woodland and Henry Wan. Aldershot, Hants: Edward Elgar, forthcoming.

Uzawa, Hirofumi. 1965. "Optimum Technical Change in an Aggregative Model of Economic Growth." *International Economic Review*. 6(1): pp 18-31.